Math 35: Chapter 10 Review

1. Solve each equation:
   a. \( x^2 = 81 \)
      \[ x = \pm 9 \]
   b. \( x^2 = 54 \)
      \[ x = \pm 3\sqrt{6} \]
   c. \( 2x^2 - 20 = 12 \)
      \[ x = \pm 4 \]
   d. \( 3x^2 + 48 = 0 \)
      \[ x = \pm 4i \]
   e. \( (x - 3)^2 = 24 \)
      \[ x - 3 = \pm 2\sqrt{6} \]
      \[ x = 3 \pm 2\sqrt{6} \]
   f. \( \sqrt{(3x + 1)^2} = 25 \)
      \[ 3x + 1 = \pm 5 \]
      \[ x = -2, \frac{4}{3} \]

2. Solve using completing the square:
   a. \( x^2 - 7x + 12 = 0 \)
      \[ x = \frac{7 + \sqrt{49 - 48}}{2} \]
      \[ x = \frac{7 + \sqrt{1}}{2} \]
      \[ x = \frac{7}{2} \]
   b. \( x^2 + 2x - 5 = 0 \)
      \[ x = \frac{-2 \pm \sqrt{4 + 20}}{2} \]
      \[ x = -1 \pm \sqrt{6} \]
   c. \( x^2 - 2x + 10 = 0 \)
      \[ x = \frac{2 \pm \sqrt{4 - 4 \cdot 10}}{2} \]
      \[ x = 1 \pm 3i \]

3. State the quadratic formula:
   \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

4. Determine if the quadratic equation has 1) Two real solutions, 2) One real solution, or 3) No real solutions. (Hint: use the discriminant)
   a. \( 2x^2 - 5x - 7 = 0 \)
      \[ b^2 - 4ac = 25 - 4 \cdot 2 \cdot (-7) \]
      \[ = 81 \]
      \[ Two \ Real \ Solutions \]
   b. \( 3x^2 + 2x = -4 \)
      \[ b^2 - 4ac = 4 - 4 \cdot 3 \cdot (-4) \]
      \[ = 48 \]
      \[ No \ Real \ Solutions \]
   c. \( x^2 + 12x + 36 = 0 \)
      \[ b^2 - 4ac = 144 - 4 \cdot 1 \cdot 36 \]
      \[ = 0 \]
      \[ One \ Real \ Solution \]

5. Solve each using the quadratic formula:
   a. \( 3x^2 + 7x = 0 \)
      \[ a = 3 \]
      \[ b = 7 \]
      \[ c = 0 \]
      \[ x = \frac{-7 \pm \sqrt{49 - 4 \cdot 3 \cdot 0}}{6} \]
      \[ = -\frac{7}{3} \]
   b. \( x^2 - 11x = -30 \)
      \[ a = 1 \]
      \[ b = -11 \]
      \[ c = 30 \]
      \[ x = \frac{11 \pm \sqrt{121 - 4 \cdot 1 \cdot 30}}{2} \]
      \[ = \frac{11 \pm \sqrt{1}}{2} \]
      \[ = \frac{11 \pm 1}{2} \]
      \[ = 0, 5 \]
   c. \( 6a^2 + a - 15 = 0 \)
      \[ a = 14 \]
      \[ b = 1 \]
      \[ c = -15 \]
      \[ x = \frac{-1 \pm \sqrt{1 + 4 \cdot 14 \cdot (-15)}}{28} \]
      \[ = \frac{-1 \pm 19}{14} \]
      \[ = -\frac{19}{14}, -\frac{1}{2} \]
d. \( x^2 - 6x + 7 = 0 \)  
\[
\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - 4(1)(7)}}{2} \\
\alpha = \frac{6 \pm \sqrt{36 - 28}}{2} \\
\alpha = \frac{6 \pm \sqrt{8}}{2} \quad x = 3 \pm \sqrt{2} 
\]

6. Solve each equation:

a. \( x = \sqrt{4x + 60} \)  
\[
x^2 = 4x + 60 \\
x^2 - 4x - 60 = 0 \\
(x - 10)(x + 6) = 0 \\
x = 10, -6 
\]

b. \( \sqrt{x^2 + 2x - 4} = x \)  
\[
x^2 + 2x - 4 = x^2 \\
2x - 4 = 0 \\
x = 2 
\]

c. \( \sqrt{x^2 - x - 12} = x + 3 \)  
\[
x^2 - x - 12 = x^2 + 6x + 9 \\
-7x = 21 \\
x = -3 
\]

e. \( x^4 - 13x^2 + 36 = 0 \)  
\[
x^2 = 9 \text{ or } x^2 = 4 \\
x = \pm 3, \pm 2 
\]

f. \( 4m^4 - 5m^2 + 1 = 0 \)  
\[
Let \ u = m^2 \\
4u^2 - 5u + 1 = 0 \\
(u - \frac{1}{4})(u - 1) = 0 \\
m = \frac{1}{4}, 1 
\]

g. \( (x^2 - 1)^2 - (x^2 - 1) - 6 = 0 \)  
\[
Let \ u = x^2 \\
u^2 - u - 4 = 0 \\
(u - 4)(u + 1) = 0 \\
x = \pm 4, -1 
\]

7. Determine each of the following questions: 1) Does the parabola open up or down? 2) Find the Vertex of the parabola, 3) State the axis of symmetry, 4) Find the x-intercept, 5) Find the y-intercept and 6) Graph the parabola.

a. \( f(x) = x^2 + 5x \)  
\[
\text{Vertex: } (-\frac{5}{2}, -\frac{25}{4}) \\
\text{Axis of Symm: } x = -\frac{5}{2} \\
\text{Opens up} \\
\text{X-int: } (0, 0), (-5, 0) \\
\text{Y-int: } (0, 0) 
\]

d. \( h(x) = (x - 3)^2 + 4 \)  
\[
\text{Vertex: } (3, 4) \\
\text{Axis of Sym: } x = 3 \\
\text{Opens up} \\
\text{X-int: None} \\
\text{Y-int: } (0, 13) 
\]

b. \( f(x) = x^2 - 2x - 8 \)  
\[
\text{Vertex: } (-1, 9) \\
\text{Axis of Sym: } x = -1 \\
\text{Opens up} \\
\text{X-int: } (-2, 0), (4, 0) \\
\text{Y-int: } (0, -8) 
\]

e. \( h(x) = -(x + 2)^2 - 3 \)  
\[
\text{Vertex: } (-2, -3) \\
\text{Axis of Sym: } x = -2 \\
\text{Opens down} \\
\text{X-int: None} \\
\text{Y-int: } (0, -7) 
\]

f. \( m(x) = 4(x + 4)^2 - 1 \)  
\[
\text{Vertex: } (-4, -1) \\
\text{Axis of Sym: } x = -4 \\
\text{Opens up} \\
\text{X-int: } (3, 5, 0), (4, 5, 0) 
\]
8. The Norco Choir is having a performance. They have estimated the income for the spring performance to be estimated by the following function

\[ I(x) = -x^2 + 22x - 30 \]

where \( x \) is the price of each ticket and \( I \) represents the income. In hundreds of dollars.

a. How much should they charge for each ticket to maximize their income?

\[
\frac{-b}{2a} = \frac{-22}{2(-1)} = \frac{-22}{-2} = 11
\]

b. What is the maximum income?

\[ I(11) = 9100 \]

9. Josh tosses a ball upward from the top of a 60 foot building. The height, \( H(t) \), of the ball at any time \( t \) can be determined by the functions

\[ H(t) = -16t^2 + 88t + 60 \]

a. At what time will the ball attain its maximum height?

\[
\frac{-b}{2a} = \frac{-88}{2(-16)} = \frac{-88}{-32} = 2.75 \text{ seconds}
\]

c. What is the maximum height?

\[ 181 \text{ feet} \]

10. Solve each inequality:

a. \((x - 3)(x + 4) > 0\) 

b. \(x^2 - 11x + 30 \geq 0\)

c. \(x^2 + x \leq -12\) 

d. \((x - 6)(x + 2)(x - 1) > 0\)